Do Estimated Taylor Rules Suffer from Weak Identification?

Juan Ignacio Urquiza
University of Houston

November 2010

Abstract

This paper explores the possibility of weak identification in estimated Taylor rule regressions with interest rate smoothing. We argue that the presence of smoothing renders the information for the estimates of Taylor rule parameters dependent on its true value in such a way that as it approaches unity inference could be spurious. In the literature, quarterly estimates of the smoothing parameter are typically in the order of 0.7 – 0.9. Therefore, we conduct a series of Monte Carlo experiments for empirically relevant sample sizes and values of the smoothing coefficient. Our results show that the actual size of a nominal 5% test is always oversized, hitting rejection rates of up to 20%. Altogether, our results suggest that evidence supporting the Taylor Principle could be spurious.

KEYWORDS: Interest Rate Smoothing; Nonlinear Least Squares; Spurious Inference; Zero-Information-Limit-Condition.

JEL CLASIFICATIONS: C12, C22, E52.

Correspondence:

Juan I. Urquiza
Department of Economics
University of Houston
Houston, TX 77204-5019

Tel (832) 528-9835
Fax (713) 743-3798
Email: jiurquiza@uh.edu
http://www.uh.edu/~jiurquiz

1. Introduction

Examples and extensions of John Taylor's (1993) monetary policy rule are now widespread in the empirical macro literature, and estimates of the Federal Reserve's responses to changes in inflation expectations and the business cycle are ubiquitous. Arguably, much of the attention received by these simple rules is due to the fact that they describe the actual behavior of the federal funds rate surprisingly well.

However, recent research by Nelson and Startz (2007) and Ma and Nelson (2010) indirectly suggests that the form of the estimated Taylor rule in a nonlinear least squares (NLS) setting may be subject to weak identification. In the literature, forward-looking versions of the Taylor rule such as the one proposed by Clarida, Galí, and Gertler (2000) were usually estimated using the generalized method of moments (GMM), which requires instruments for the endogenous regressors.[1] However, after Orphanides' (2001, 2004) argument on the operational implementation of such rules which use ex-post data that was not available to policy makers in real time, the use of real-time data has become industry standard. This alleviates the need for instruments and allows Taylor rules to be estimated by least squares. This paper investigates whether Taylor rules suffer from weak identification in the presence of interest rate smoothing.

In his seminal paper, Taylor (1993) proposed a feedback policy rule of the form:

$$r_t^* = \pi_t + \delta(\pi_t - \pi^*) + \omega\hat{y}_t + R^*, \tag{1}$$

where r_t^* is the target interest rate, π_t is the inflation rate, π^* is the target level of inflation, $\hat{y}_t$ is the output gap, and R^* is the long-run equilibrium real interest rate. Intuitively, this rule implies

[1] Mavroeidis (2010) examines the possibility of weak identification of Taylor rules in the context of a dynamic stochastic general equilibrium (DSGE) model and GMM estimation.

that the Fed raises the nominal interest rate when inflation rises above its target or when output is above its potential output.

If we combine the parameters π^* and R^* into a single constant term, we can rewrite equation (1) as follows:

$$r_t^* = \mu + (1 + \delta)\pi_t + \omega\hat{y}_t. \tag{2}$$

The condition that $(1 + \delta) > 1$ is referred to as the Taylor Principle, and it implies that the Fed rises the nominal interest rate by more than point-for-point when inflation rises above target, so that the real interest rate rises. Notice that this condition has important stability connotations: if the Taylor Principle is satisfied and the Fed rises the nominal interest rate by more that point-for-point so that the real interest rate increases, this leads to a negative output gap and, in turn, to a decrease in inflation.

Clarida, Galí, and Gertler (2000) propose a forward-looking version of the simple Taylor rule. In particular, they consider an extension in which the Fed responds to expectations of current and future inflation:

$$r_t^* = \mu + (1 + \delta)E_t\pi_{t+h} + \omega\hat{y}_t, \tag{3}$$

where $E_t\pi_{t+h}$ is the expectation of the inflation rate at time $t+h$ formed at time t. As argued by Orphanides (2001, 2004), in order for this policy rule to be operational, it requires information to be contemporaneously available to policy makers at the time decisions are made. Therefore, he promotes the use of real-time data which permits estimation of the parameters by nonlinear least squares. The use of real time data, and estimation of Taylor rules by NLS is now widespread.

Furthermore, researchers typically allow for partial adjustment or interest rate smoothing. The basic idea is that the federal funds rate may not adjust instantaneously towards its target level but rather in a gradual fashion. Therefore, we follow the literature in allowing for AR(1) smoothing so that the actual federal funds rate can be thought as:

$$r_t = \rho r_{t-1} + (1-\rho) r_t^* + \varepsilon_t, \tag{4}$$

Where ρ reflects the degree of smoothing. That is, the faster the response to shocks, the closer this parameter approaches to zero.

Therefore, if we substitute equation (3) into (4), we arrive at the typical specification for the nominal interest rate:

$$r_t = \rho r_{t-1} + (1-\rho)[\mu + (1+\delta) E_t \pi_{t+h} + \omega \hat{y}_t] + \varepsilon_t. \tag{5}$$

Consistent with a very slow adjustment, estimates of ρ for quarterly data are usually in the order of 0.7 – 0.9. However, we argue that the presence of smoothing renders the information for the estimates of the Taylor rule parameters dependent on ρ in such a way that as it approaches unity inference may be spurious. Nelson and Startz (2007) refer to this phenomenon as the Zero-Information-Limit-Condition (ZILC).

The rest of the paper is organized as follows. In section 2, we present the basic model studied by Nelson and Startz (2007) and we summarize their main results. Section 3 examines smoothing by introducing an intermediate model that allows us to isolate its effects. In section 4 we augment the nonlinear model of Nelson and Startz (2007) to resemble a Taylor rule with interest rate smoothing. We demonstrate that the ZILC holds for nonlinear regression models with smoothing, in particular for empirically relevant sample sizes and values of the smoothing parameter. Tests at the nominal 5% level are almost always oversized if $1+\delta=1$, *i.e.* when the Taylor Principle does not hold. Section 5 concludes.

2. Nelson and Startz (2007): Overview

Consider the following nonlinear regression model studied by Nelson and Startz (2007), hereafter NS, where β is the parameter of interest:

$$y_t = \gamma(x_t + \beta z_t) + \varepsilon_t \tag{6}$$

The model is identified if $\gamma \neq 0$, and NS focus on the behavior of $\hat{\beta}$ as γ approaches zero. NS show that for Normal errors, γ controls the amount of information about β in the data for a given sample size. In particular, the asymptotic variance of $\hat{\beta}$ is proportional to γ^{-2}. They demonstrate that as γ approaches zero, the information contained in $\hat{\beta}$ goes to zero, and its variance diverges. If the regressors are standardized to have unit variance and correlation ρ, the asymptotic variance of $\hat{\beta}$ is given by the following expression:

$$\mathrm{var}(\hat{\beta}) = \left(\frac{\sigma^2}{T}\right)\left(\frac{1}{\gamma^2}\right)\frac{1+2\beta\rho+\beta^2}{(1-\rho^2)}. \tag{7}$$

In the limiting case that $\gamma = 0$ the variance of $\hat{\beta}$ becomes infinite. NS refer to this as the Zero-Information-Limit-Condition (ZILC) for $\hat{\beta}$. The information for $\hat{\beta}$, *i.e.*, the inverse of its variance, is proportional to γ^2. Since $E(\hat{\gamma})^2 = \gamma^2 + \mathrm{var}(\hat{\gamma})$ if $\hat{\gamma}$ is unbiased, and the asymptotic variance of $\hat{\gamma}$ for standardized regressors becomes:

$$\mathrm{var}(\hat{\gamma}) = \left(\frac{\sigma^2}{T}\right)\frac{1}{(1-\rho^2)}, \tag{8}$$

there will be an upward bias in estimating γ^2 when γ is close to zero. This will cause the variance of $\hat{\beta}$ to be downward biased. A natural measure of this overestimation of γ^2 is $E(\hat{\gamma}^2)/\gamma^2$. To focus on how this ratio affects inference, we present some main results from NS for the case of $\beta = 0$ with standardized and uncorrelated regressors, and include $E(\hat{\gamma}^2)/\gamma^2$ for the various cases of γ and T that they consider, and report them in Table 1.[2]

NS pay particular attention to the case when T = 100 and $\gamma = 0.01$, so that the model is identified, but only weakly. In this case $E(\hat{\gamma}^2)/\gamma^2 = 101$, estimated information for $\hat{\beta}$ is twenty times larger than asymptotic information, and the estimated standard error is roughly four and a half times too small. Despite the spuriously precise standard error, empirical size of the t-test at the nominal 5% level is 0.001. In addition to these problems when $\gamma = 0.01$, the convergence of $\hat{\beta}$ and its t-statistic to their asymptotic distributions is quite slow. When T = 100,000 the standard error of $\hat{\beta}$ is only slightly underestimated, which is reflected by the fact that the ratio $E(\hat{\gamma}^2)/\gamma^2$ drops to 1.1, but the empirical size of the t-test is roughly 1%. Even when T = 1,000,000, the actual size is only 3.1%, demonstrating how slowly asymptotic theory kicks in.

The phenomenon of understated standard errors and small test size, which is most apparent when T = 100 and $\gamma = 0.01$, depends on the correlation, zero in this case, between x_t and z_t. For

[2] Most of these results appear in Table 2 of Nelson and Startz (2007).

this particular pair of T and γ, the t-test is undersized for values of $\rho \in [-\frac{2}{3}, \frac{2}{3}]$, and oversized outside of this range.

The highest value of $E(\hat{\gamma}^2)/\gamma^2$ reported by NS for uncorrelated regressors is for $\gamma = 0.01$ and $T = 100$, and is again 101. It is obvious from equation (8) that as the correlation between regressors increases, the variance of $\hat{\gamma}$ increases. With 101 as a benchmark, this ratio is approximately 134, 230, and 527 for correlations of 0.5, 0.75, and 0.9 respectively, with the last two correlations resulting in oversized t-tests.

In the following two sections, we examine the extent to which more general models with smoothing are subject to weak identification and spurious inference.

3. Simple Smoothing: An Intermediate Step

In order to isolate the effect of smoothing, we first consider the following intermediate model, which we refer to as simple smoothing:

$$y_t = (1-\gamma) + \gamma(x_t + \beta z_t) + \varepsilon_t. \tag{9}$$

Notice that this model does not allow for a lagged dependent variable yet. In this case, the effect of smoothing is to lower the variance of $\hat{\gamma}$ to:

$$var(\hat{\gamma}) = \left(\frac{\sigma^2}{T}\right)\frac{1}{(2-\rho^2)}. \tag{10}$$

Overall, the effect of smoothing is to reduce the upward bias of $(\hat{\gamma}-\gamma_0)^2$, or equivalently the ratio $E(\hat{\gamma}^2)/\gamma^2$. In Table 2 we repeat a version of NS Table 2 for this simple smoothing model. The worst case benchmark from the basic NS model reduces to 51 from 101. For correlations of 0.5, 0.75, and 0.9, the ratio takes on the values 58, 71, and 85, respectively. It is apparent that the presence of smoothing should mitigate the problem of underestimating the standard error of $\hat{\beta}$, and lead to tests with less size distortions than in the basic NS nonlinear model.

4. A Nonlinear Regression Model with Smoothing

In order to determine whether estimated Taylor rules suffer from weak identification, we extend the model of NS to resemble a Taylor rule regression with interest rate smoothing. For simplicity, we consider the case in which the Fed focuses only on expected inflation while ignoring business cycle considerations. We can think of a such a model as follows:

$$y_t = (1-\gamma)y_{t-1} + \gamma\beta z_t + \varepsilon_t. \tag{11}$$

where the parameters in equation (5) are renamed so as to follow the notation of NS: $\rho = (1-\gamma)$, $(1+\delta) = \beta$, and $\mu = 0$.

Some differences between the models are worth noting. γ appears twice on the right hand side of the regression and we have lagged y_t as a regressor, both due to smoothing. Unlike the model of NS, we do not need x_t to identify γ, so it has been dropped to keep the model tractable. Dropping x_t leaves only one right hand side correlation to consider; that between z_t and y_{t-1}.

With Normal errors, it can be shown that the Information matrix for $\hat{\beta}$ and $\hat{\gamma}$ is equal to:

$$I\left(\hat{\beta},\hat{\gamma}\right)=\begin{bmatrix} \frac{\gamma^2}{\sigma^2}Tm_{zz} & \left(-\frac{\gamma}{\sigma^2}Tm_{yz}+\frac{\gamma\beta}{\sigma^2}Tm_{zz}\right) \\ \left(-\frac{\gamma}{\sigma^2}Tm_{yz}+\frac{\gamma\beta}{\sigma^2}Tm_{zz}\right) & \left(\frac{1}{\sigma^2}Tm_{yy}+\frac{\beta^2}{\sigma^2}Tm_{zz}-\frac{2\beta}{\sigma^2}Tm_{yz}\right) \end{bmatrix}, \tag{12}$$

with determinant $\Delta=\frac{\gamma^2}{\sigma^4}T^2\left(m_{zz}m_{yy}-m_{yz}^2\right)$. m_{zz} is the 2nd sample moment of z_t, etc. This implies the following asymptotic variances:

$$\text{var}(\hat{\beta})=\left(\frac{\sigma^2}{T}\right)\left(\frac{1}{\gamma^2}\right)\frac{m_{yy}-2\beta m_{yz}+\beta^2 m_{zz}}{(m_{zz}m_{zz}-m_{yz}^2)}, \tag{13}$$

and

$$\text{var}(\hat{\gamma})=\left(\frac{\sigma^2}{T}\right)\frac{m_{zz}}{(m_{zz}m_{yy}-m_{yz}^2)}. \tag{14}$$

If we standardize the regressors, so that $m_{yy}=m_{zz}=1$ and $m_{yz}=\rho$, where ρ is the correlation between z_t and y_{t-1}, we can rewrite the asymptotic variances as:

$$\text{var}(\hat{\beta})=\left(\frac{\sigma^2}{T}\right)\left(\frac{1}{\gamma^2}\right)\frac{1-2\beta\rho+\beta^2}{(1-\rho^2)}, \tag{15}$$

and

$$\text{var}(\hat{\gamma})=\left(\frac{\sigma^2}{T}\right)\frac{1}{(1-\rho^2)}. \tag{16}$$

Notice that stationarity of the dependent variable implies an additional restriction for σ^2. This becomes apparent when we take the variance on both sides of equation (11):

$$\text{var}(y_t) = (1-\gamma)^2 \text{ var}(y_{t-1}) + \gamma^2\beta^2 \text{ var}(z_t) + 2(1-\gamma)\gamma\beta Cov(y_{t-1}, z_t) + \sigma^2. \tag{17}$$

For the model to be stationary, we need that $\text{var}(y_t) = \text{var}(y_{t-1})$. Therefore, we can solve for σ^2 in terms of the other parameters so that:

$$\sigma^2 = 1-(1-\gamma)^2 - \gamma^2\beta^2 - 2(1-\gamma)\gamma\beta\rho. \tag{18}$$

If we substitute for σ^2 into the equations from above, we get the following expressions:

$$\text{var}(\hat{\gamma}) = \left(\frac{1-(1-\gamma)^2 - \gamma^2\beta^2 - 2(1-\gamma)\gamma\beta\cdot\rho}{T}\right)\frac{1}{(1-\rho^2)}, \tag{19}$$

and

$$var(\hat{\beta}) = \left(\frac{1-2\beta\rho+\beta^2}{T\cdot(1-\rho^2)}\right)\left[\frac{2-\gamma-\gamma\beta^2-2(1-\gamma)\beta\rho}{\gamma}\right]. \tag{20}$$

We can now begin to assess whether our model suffers from the ZILC. In particular, if we follow NS and take the limit of the variance of $\hat{\beta}$ with respect to γ as it approaches zero, we can confirm our intuition that this nonlinear model with smoothing is affected by the ZILC. That is,

$$\lim_{\gamma\to 0} var(\hat{\beta}) \to \infty \;\; if \;\beta\rho \neq 1. \tag{21}$$

Only in the extreme case in which $\beta\rho = 1$ is our model not subject to the ZILC.

Next, we focus on the variance of $\hat{\gamma}$. Two things are worth noticing. First, the asymptotic variance depends on the true value of γ, just as the OLS intuition would suggest for the simple AR(1) model; *i.e.*, $\hat{\gamma}$ is non pivotal. Second, the limit of this expression collapses to zero as γ approaches the ZILC point; *i.e.*, $\hat{\gamma}$ is super-consistent.

We now consider a series of Monte Carlo experiments in order to determine the effect of γ and the sample size on the actual size of a nominal 5% test. In particular, we simulate data according to equation (11) for the case of uncorrelated regressors with unit variance, and setting β equal to zero. Table 3 presents the results for different values of γ and T.[3]

In the first experiment, we set $\gamma = 0.99$ and $T = 100$ so that the model is well-identified. As one would expect, the median of estimated information and standard errors across simulations are close to asymptotic values and the size of the t-test is correct. Fixing the sample size, when we reduce γ to 0.1, while estimated information and standard errors are still close to their asymptotic counterparts, the size of the t-test is seventeen times too small. This finding is consistent with NS, who find for this pair of γ and T, there is offsetting co-variation in the numerator and the denominator of the test statistic. Large values of the numerator are accompanied by large values of the denominator, and vice-versa, so that the resulting test statistic is always small. However, this phenomenon disappears as the sample size increases. In particular, we find that when $T = 1000$ the size of the t-test is approximately right. Finally, we set γ to 0.01 and the effect of the ZILC becomes apparent: estimated information is too large, the estimated standard error is too small, and the size of the t-test is fifty times too small. In this case, the problem persists even for $T = 10{,}000$ demonstrating that asymptotic theory takes hold very slowly for this model. Only with T = 100,000 observations does the size of the test match nominal size.

We now consider the effect of correlation between the regressors on test size. Figure 1 shows the empirical size of a nominal 5% test using different values of γ and T. Using T = 100,

[3] For this nonlinear regression model with smoothing we abandon the metric $E(\hat{\gamma}^2)/\gamma^2$ as $\hat{\gamma}$ is no longer unbiased.

$\gamma = 0.01$, and $\beta = 0$ as our benchmark case, Panel C of Figure 1 shows that the t-statistic is undersized for low correlations and oversized for high correlations, with rejection rates of up to 50% in extreme cases. The extreme rejection rates are not as high as in NS, and this is due to the effect of smoothing on the variance of $\hat{\beta}$. As T is increases to 1000 in Panel D, size is reduced, but the basic NS size "smile" is still present. In panel A, where $\gamma = 0.1$ and T = 100, the increase in γ leaves the basic message unchanged; the test is undersized for low correlations, and oversized for high correlations. When T is increased to 1000 in Panel B, the test is approximately correctly sized for most correlation values.

For their nonlinear model, NS focused on the case where $\beta = 0$. For our nonlinear smoothing model, if we are to think of it as a Taylor rule regression, a more empirically relevant value of β is unity. In this case, the Federal Reserve changes the nominal rate point-for-point in response to changes in inflation, leaving the real interest rate unchanged. If $\beta < 1$, the Fed allows the real interest rate to fall when inflation increases, and if $\beta > 1$ the Fed increases the nominal rate by more than point-for-point, increasing the real interest rate, and satisfying the Taylor Principle.

In many cases of estimated Taylor rules, the null hypothesis of $\beta = 1$ is rejected in favor of the alternative that the Fed follows the Taylor Principle. In order to assess whether or not these rejections are the result of weak identification of Taylor rule regressions, we generate artificial data for empirically relevant values of γ and T, and setting $\beta = 1$. The results for uncorrelated regressors with unit variance are presented in Table 4.

In the first experiment, we set γ to 0.5 and T = 50 so that the model is identified. Consistently, the size of the t-test is approximately correct. Fixing the sample size, when we reduce the value of γ to 0.2 the problem begins to manifest itself. While estimated information and standard errors are close to asymptotic values, the size of the t-test is roughly two and a half times too high. The problem persists even for T = 250, and size only converges with T = 1000 – a sample size that is unrealistically high if we are to think of estimating monetary policy rules. As we further reduce the value of γ to 0.1, the ZILC appears to hold: estimated information is too large, standard errors too small, and the size of the t-test is three times too high. Only when T = 1000 does the size of the test match nominal size. Finally, we consider the extreme case of $\gamma = 0.01$and T = 50. In this case, estimated information is four times larger than asymptotic information, standard errors are two times too small, and the rejection rate increases to 20%. Notice that for this small value of γ, the problem persists even for T = 1000!

Figures 2 – 5 show the effect of correlation on the size of the t-test for different values of γ and T. We should note here that the size "smile" of NS does not generalize to the case of $\beta = 1$. While we do not report the results here, the size curve of the NS nonlinear regression model for non-zero values of β is often completely above the 5% line, especially for values of β greater than unity. This finding is consistent with our results, as the t-test appears to be oversized over the whole range of correlations when the ZILC is met.

In Figure 2, $\gamma = 0.5$ so that the model is identified. Consistently, we find that that $\hat{\beta}$ does not suffer from ZILC when T = 50. Moreover, the size of the t-test is almost 5% for virtually all correlations.

In Figure 3, $\gamma = 0.2$, which corresponds to a smoothing coefficient of 0.8, and the sample size ranges from 50 to 1000. For T = 50 and 100, we see that the size of the t-test is always too high, often twice nominal size. For T = 250, the problem begins to disappear, and for T = 1000 size is approximately correct except for extreme negative correlations.

In Figure 4, $\gamma = 0.1$, and the problem worsens. For T = 50 and 100, empirical size is now much higher than nominal size, often 3 to 4 times too large. Again, the problem begins to mitigate for T = 250, and is almost gone for T = 1000.

In Figure 5, we consider the extreme case that $\gamma = 0.01$. Here the problem is quite serious. For sample sizes of 50 and 100, empirical size routinely exceeds 20%. Even with 1000 observations, test size is rarely below 10%.

Altogether, our results suggests that an econometrician estimating Taylor rules with interest rate smoothing may spuriously find evidence supporting the Taylor Principle as t-tests appear to be oversized for empirically relevant sample sizes and values of the smoothing parameter.

5. Conclusion

This paper examines whether estimated Taylor rules suffer from weak identification in a nonlinear least squares setting. To address this issue, we extend the basic nonlinear model of Nelson and Startz (2007) to resemble a Taylor rule regression with interest rate smoothing. We demonstrate that the Zero-Information-Limit-Condition holds for this model; in particular, we show that the inverse of the asymptotic variance of Taylor rule estimates goes to zero as the smoothing parameter approaches unity. Quarterly estimates of this parameter are typically within the range 0.7 – 0.9, which suggests that estimated Taylor rules could be affected by weak identification.

We complement this analysis with a series of Monte Carlo experiments. In particular, we generate artificial data according to the nonlinear regression model with smoothing presented in section 4, and fixing the parameter of interest – i.e., that resembling the Taylor Principle – to unity. This allows us to assess whether or not the rejections of the null hypothesis that this parameter equals unity are spurious. Overall, our results show that for empirically relevant sample sizes and values of the smoothing parameter, estimated information is too large, the standard errors too small, and t-statistics oversized.

References

Clarida, R., J. Galí, and M. Gertler (2000). "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." Quarterly Journal of Economics **115**(1): 147-180.

Nelson, C. R., and R. Startz (2007). "The Zero-Information-Limit Condition and Spurious Inference in Weakly Identified Models." Journal of Econometrics **138**: 47-62.

Ma, J., and C. R. Nelson (2010). "Valid Inference for a Class of Models Where Standard Inference Performs Poorly; Including Nonlinear Regression, ARMA, GARCH, and Unobserved Components." working paper, University of Washington.

Mavroeidis, S. (2010). "Monetary Policy Rules and Macroeconomic Stability: Some New Evidence." American Economic Review **100**(1): 491-503.

Orphanides, A. (2001). "Monetary Policy Rules Based on Real-Time Data." American Economic Review **91**(4): 964-985.

Orphanides, A. (2004). "Monetary Policy Rules, Macroeconomic Stability, and Inflation: A View from the Trenches." Journal of Money, Credit, and Banking **36**(2): 151-175.

Taylor, J. B. (1993). "Discretion versus Policy Rules in Practice." Carnegie-Rochester Conference Series on Public Policy **39**(1): 195-214.

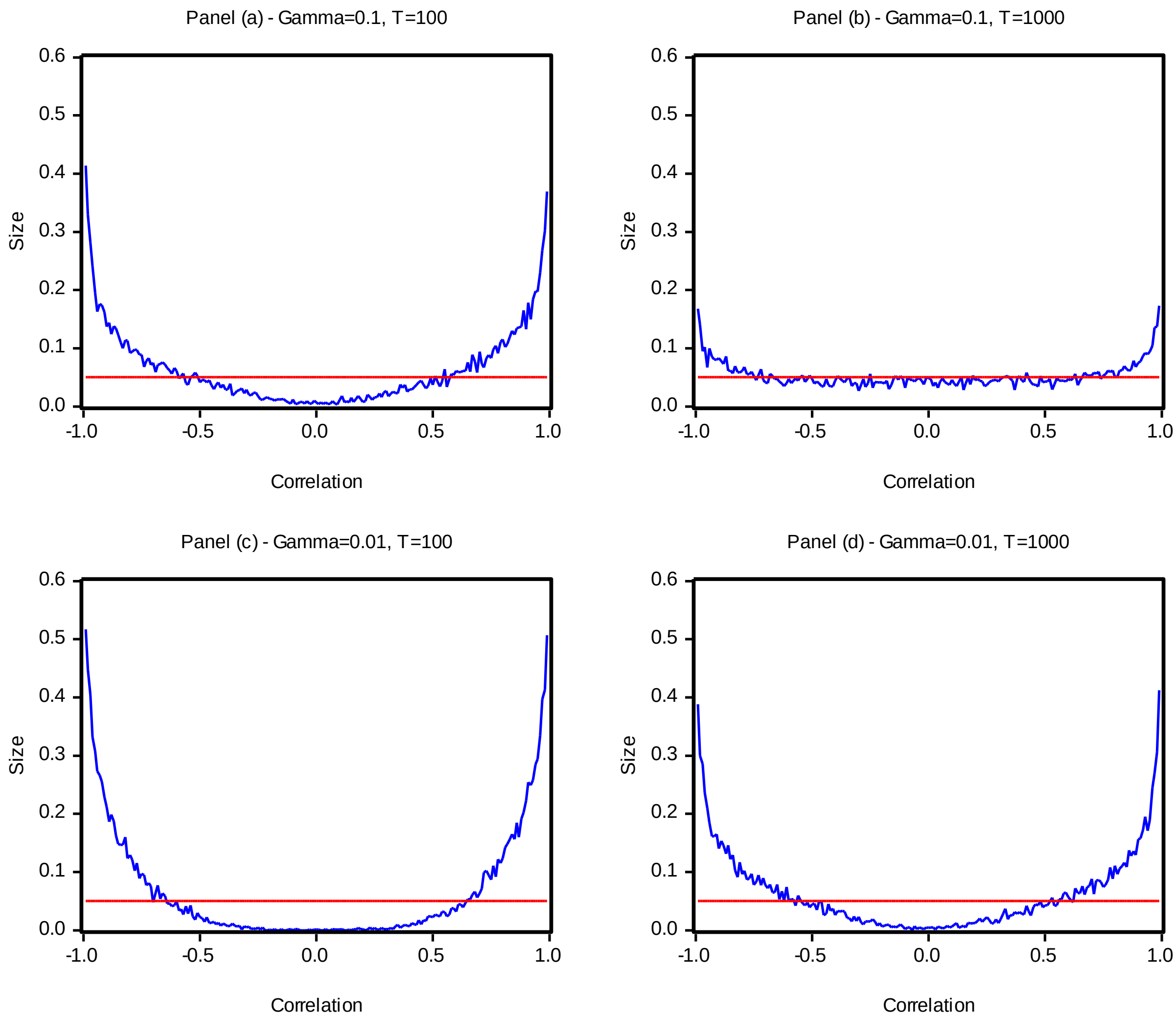

Figure 1 - Actual Size of a Nominal 5% Test, β=0.

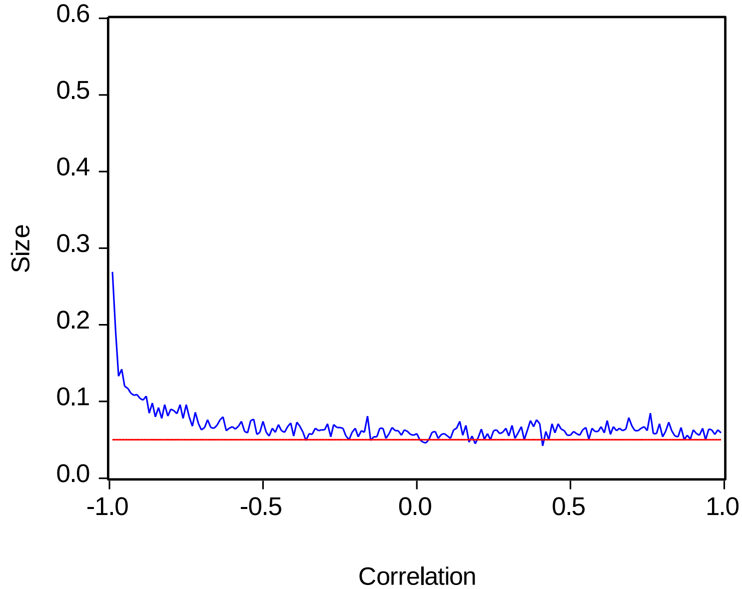

Figure 2 - Actual Size of a Nominal 5% Test, γ=0.5, β=1, and T=50.

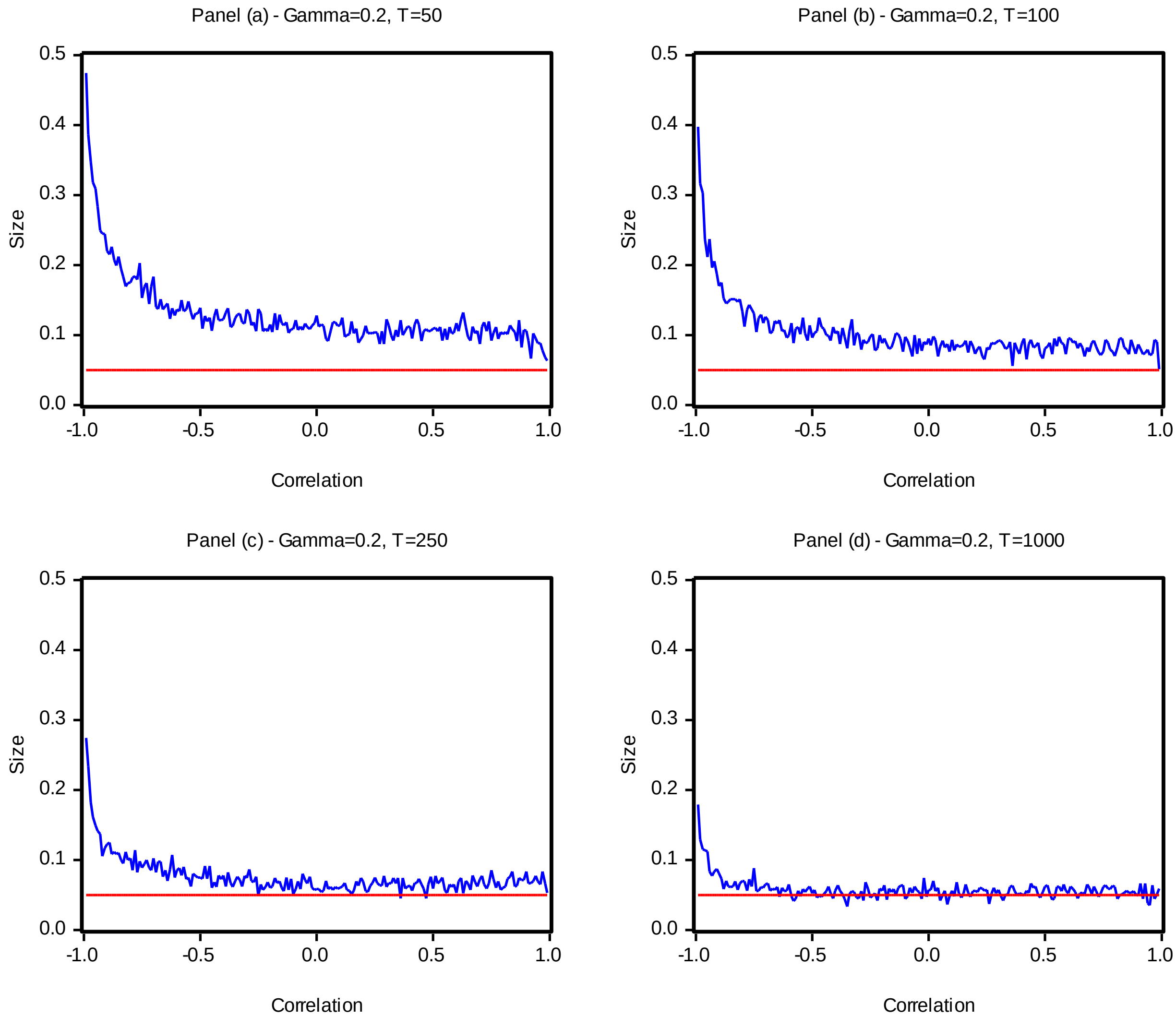

Figure 3 - Actual Size of a Nominal 5% Test, γ=0.2 and β=1.

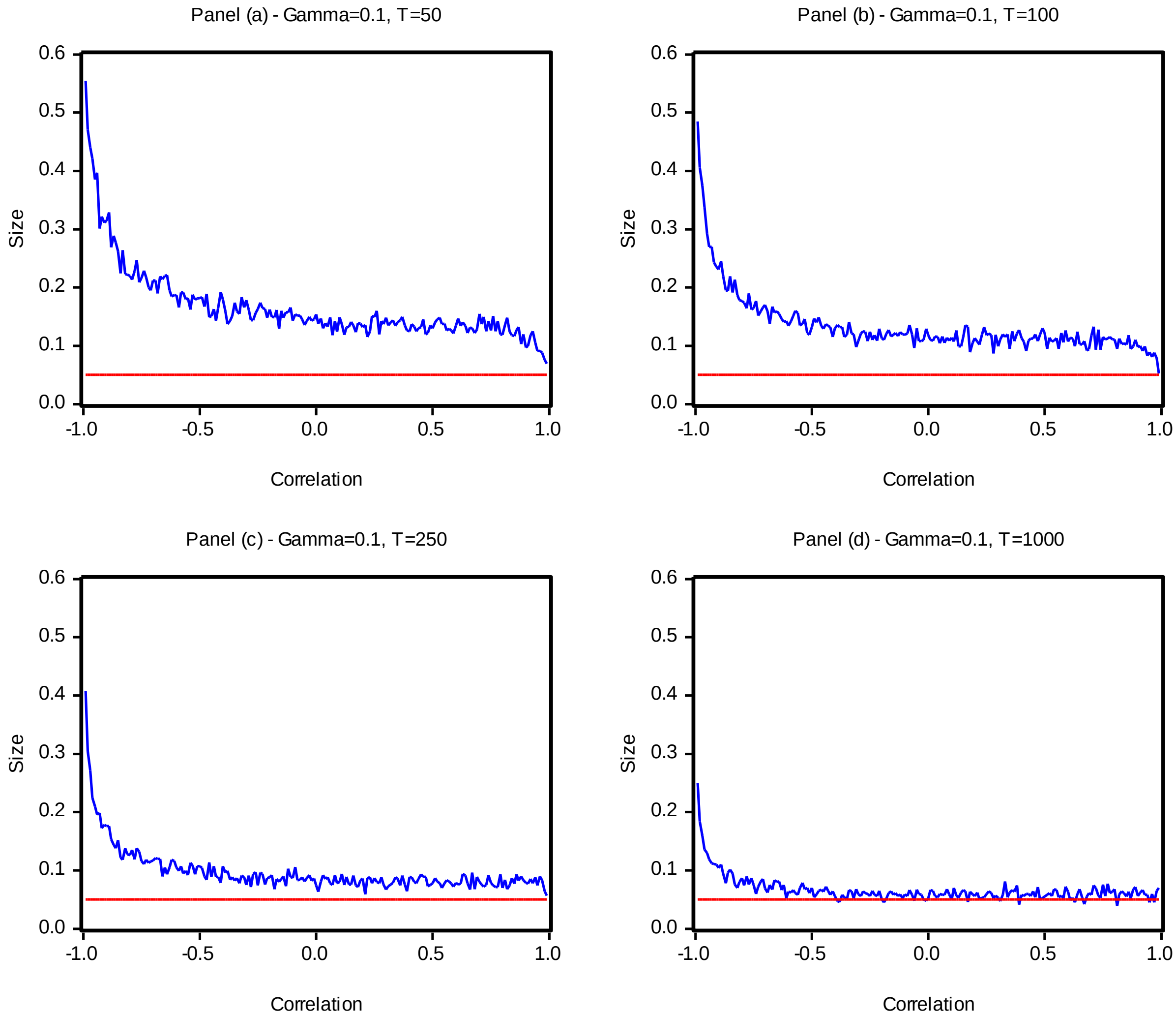

Figure 4 - Actual Size of a Nominal 5% Test, γ=0.1 and β=1.

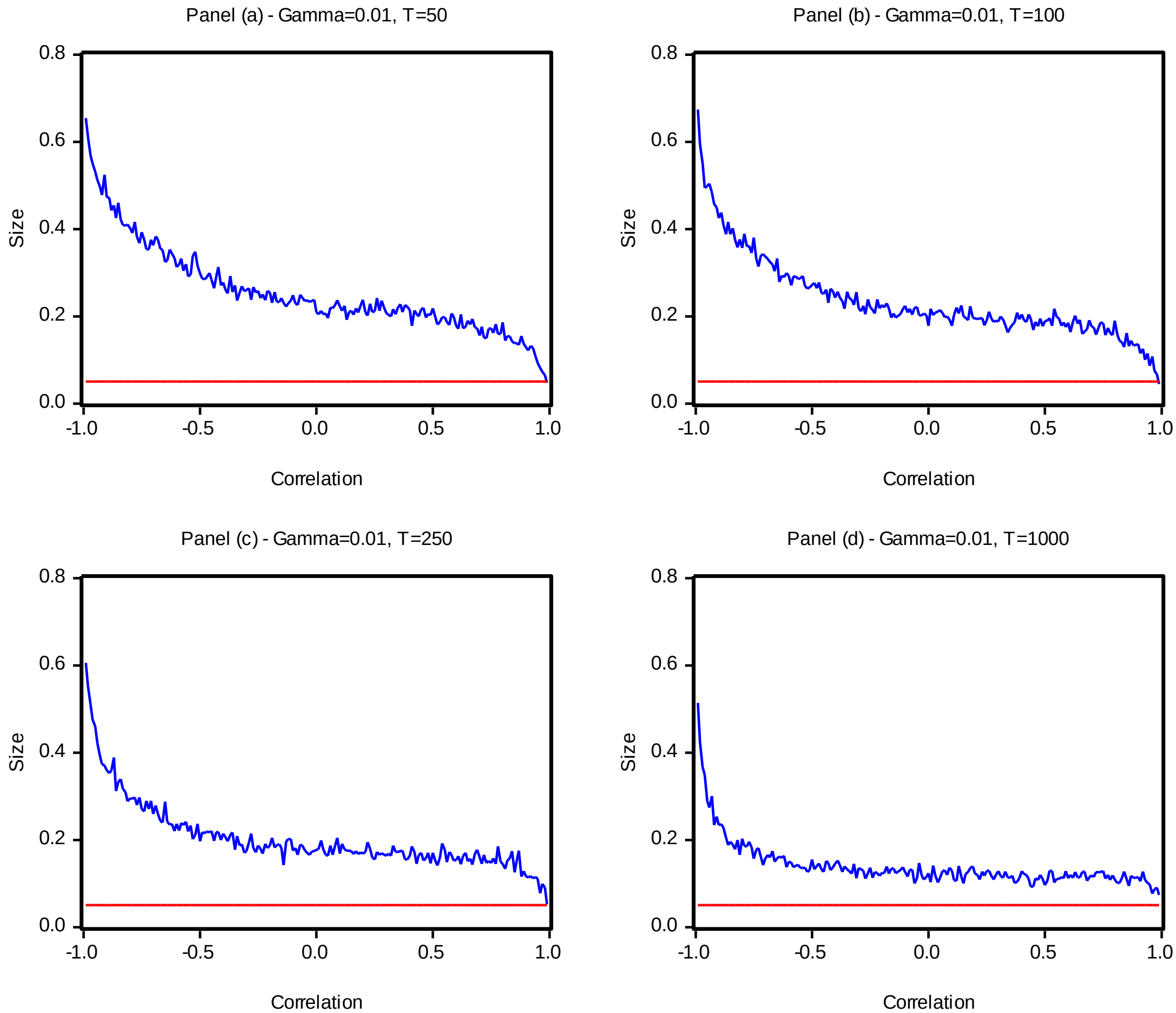

Figure 5 - Size of a Nominal 5% Test, γ=0.01 and β=1.

Table 1 – Nelson and Startz Model

Sampling distributions for non-linear regression ($\beta = 0 , \rho = 0$)							
γ	T	Information $I_{\hat{\beta}}$		Standard error of $\hat{\beta}$		Freq. $\lvert t_\beta \rvert > 1.96$	$E(\hat{\gamma}^2)/\gamma^2$
		Asy.	Median	Asy.	Median		
1	100	100	108	0.10	0.10	0.048	1.01
0.1	100	1	0.88	1	1.32	0.001	2
0.1	1000	10	9.30	0.32	0.33	0.016	1.10
0.1	10,000	100	100	0.10	0.10	0.045	1.01
0.01	100	0.01	0.20	10	1.95	0.001	101
0.01	1000	0.10	0.26	3.16	1.97	0.000	11
0.01	10,0000	10	9.05	0.32	0.33	0.018	1.10
0.01	1,000,000	100	100	0.10	0.10	0.053	1.01

Table 2 – Simple Smoothing Model

Sampling distributions for non-linear regression ($\beta = 0, \rho = 0$)							
γ	T	Information $I_{\hat{\beta}}$		Standard error of $\hat{\beta}$		Freq. $\lvert t_\beta \rvert > 1.96$	$E(\hat{\gamma}^2)/\gamma^2$
		Asy.	Median	Asy.	Median		
1	100	100	99	0.10	0.10	0.054	1.01
0.1	100	1	0.81	1	1.11	0.001	1.50
0.1	1000	10	9.61	0.32	0.32	0.025	1.05
0.1	10,000	100	99.29	0.10	0.10	0.047	1.01
0.01	100	0.01	0.09	10	3.26	0.001	51
0.01	1000	0.10	0.14	3.16	2.67	0.000	6
0.01	10,0000	10	10.03	0.32	0.32	0.042	1.05
0.01	1,000,000	100	99.54	0.10	0.10	0.031	1.01

Table 3 – Nonlinear Regression Model with Smoothing

Sampling distributions for non-linear regression $(\beta = 0, \rho = 0)$						
γ	T	Information $I_{\hat{\beta}}$		Standard error of $\hat{\beta}$		Freq. $\lvert t_{\beta}\rvert > 1.96$
		Asy.	Median	Asy.	Median	
0.99	100	98.02	94.76	0.10	0.10	0.052
0.1	100	5.26	5.18	0.44	0.44	0.003
0.1	1000	52.63	51.44	0.14	0.14	0.049
0.01	100	0.50	0.82	1.41	1.10	0.001
0.01	1000	5.03	4.94	0.45	0.43	0.004
0.01	10,000	50.25	50.56	0.14	0.14	0.036
0.01	100,000	502.51	502.17	0.04	0.04	0.049

Table 4 – Nonlinear Regression Model with Smoothing

Sampling distributions for non-linear regression ($\beta = 1, \rho = 0$)						
γ	T	Information $I_{\hat{\beta}}$		Standard error of $\hat{\beta}$		Freq. $\lvert t_{\beta} \rvert > 1.96$
		Asy.	Median	Asy.	Median	
0.5	50	12.50	11.67	0.28	0.29	0.054
0.2	50	3.13	3.39	0.57	0.54	0.128
0.2	100	6.25	6.59	0.40	0.39	0.095
0.2	250	15.63	15.83	0.25	0.25	0.058
0.2	1000	62.50	63.17	0.13	0.13	0.057
0.1	50	1.39	1.74	0.85	0.76	0.154
0.1	100	2.78	3.31	0.60	0.55	0.118
0.1	250	6.94	7.37	0.38	0.37	0.074
0.1	1000	27.78	27.93	0.19	0.19	0.052
0.01	50	0.13	0.56	2.81	1.34	0.209
0.01	100	0.25	0.60	1.99	1.29	0.178
0.01	250	0.63	1.16	1.26	0.93	0.177
0.01	1000	2.53	3.38	0.63	0.54	0.123